Results and Impacts of European eLearning in Mathematics

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Abstract— 20 years ago the computer algebra system Mathematica was developed by Wolfram Research in the United States. For the first time symbolic mathematics was easy to handle on a computer. Together with the development of markup languages for the Internet this has given us the possibility to develop high-quality digital content for eLearning in mathematics. Many European projects into mathematical eLearning have had especially one main goal: To prevent the number of students in mathematical courses and the number passing the examination reaching a critical low level. To meet this challenge the pedagogics and methods of communicating mathematics will have to undergo an innovative process where new technology is important. However this has turned out to be harder than expected, the main reason being that it requires expertise in several subject matters as well as expertise in several technologies. In the case of mathematics, the required areas of expertise include that of professional mathematicians, software engineers, publishers, and perhaps learning theorists. This article presents an overview of the most important European projects into online learning in mathematics in the context of the Bologna Declaration concerning the tayloring of the structure of the European Higher Education Area (EHEA).

Keywords— Mathematical eLearning, Online Calculations, Pedagogical Value, SciWriter, Semantic Markup, Mathematical Databases, Xmath eBook, Steplets, Multilingual and Multicultural eContent, Mathematical Grammars, OpenMath, TextMath Editor, WebALT Editor, The European Workshop

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1. Introduction

MATHEMATICAL education has traditionally two main goals: to train students in logical and abstract thinking and give students an ability to use mathematical knowledge in fields other than mathematics itself. To achieve especially the last goal will require what we call understanding and the understanding of understanding will be important [1]. We need to let the student feel importance in a wider context than the upcoming examination. The use of eLearning and interactive and event-driven documents may enhance understanding and allow an explorative and personal way of working. These documents are efficient and direct and may combine different kinds of non-linear hypermedia. This is expected to enhance mathematical pedagogics on the Internet and will stimulate activity, independence and collaboration and may also prevent sharp distinctions between boys and girls using ICT. In this way the teacher will be more of an instructor than a professor.

2. The Xmath project

Xmath [2] was the first EU project focusing on mathematical markup in the devolepment of mathematical eLearning resources in 2001. This project, connected to the EU Minerva Action [3], was coordinated by Buskerud University College [4] in Norway and lasted for 2 years (2001-2003).
The objectives of Xmath was first of all to design a framework making it possible to use the markup MathML in mathematical education on the Internet and making it possible to evaluate the pedagogical advantages of netbased education in mathematics. MathML is an application of XML for describing mathematical notation and capturing. It is a recommendation of the World Wide Web Consortium [5] and includes markup for both presentation and content. Using content markup, formulae and expressions may be exported to a computer algebra system.
One of the key elements of netbased pedagogics generally is connected to the communication process student-student and student-teacher over distance (synchronous and asynchronous). The framework then had to include a management system and communication

resources understanding MathML documents. Xmath developed the very first system for e-communication using mathematical symbols, operators and shapes (ScientificTalk). The framework also had to include software making it possible to generate MathML code automatically and to evaluate (calculate) content markup. The system WebMathematica [6] was introduced by Wolfram Research in 2001 and was an important tool in the Xmath project used to develop the Xmath eBook [7].

To demonstrate the use of MathML and to be able to evaluate the pedagogical impact of using it, a pilot course in university mathematics had to be developed [8]. Since MathML was not generally well known because this in 2001 was a brand-new technology or even more important in 2001 the lack of browsers to render it, the project partners had to take responsibility in connection with dessimination of both MathML itself and the envisaged outputs of the project. It was expected that the use and interest for MatmML would be increasing both in education and publishing. The project group had as an objective to create and maintain virtual rooms concerning the use of MathML and WebMathematica in education and publishing.

The most important results of Xmath are the Pilot Course and the Xmath eBook both used and evaluated in practical teaching. Another important result is the European Workshop on MathML and Scientific eContents now in 2008 replacing "MathML" with "Mathematical" [9].

The Pilot Course was designed using MathML presentation markup and evaluated in distance education at Sogndal University College [10]. The Pilot Course covers number theory, functions, analysis and calculus divided into 3-4 levels and is supposed to be a supplement to a common textbook. Most students agreed about good relevance to the mathematics courses and on a good structure of the book. Online eBooks have several advantages: dynamic content, easy navigation and links to other resources on the semantic web, e.g. using an online mathematics course in one window and your favorite mathematics workplace in a second.

The Xmath eBook is composed of what we call Steplets [11]. A Steplet is a routine for solving arbitrary mathematical problems into specific fields step-by-step using webMathematica. The Steplet algorithms developed in Xmath have been praised by the students and are especially important in distance edication where the teacher is absent. As stated by a student at Buskerud University College in a compulsory project report: "*Steplets are moving focus from detailed calculating techniques to understanding mathematics*".

The European Workshop on Mathematical and Scientific eContents has been arranged 4 times and the 5th workshop already planned for Salamanca in Spain in 2010. The workshop focuses on the didactics and technologies related to web-based interactive mathematics, math typesetting, eLearning and scientific communication. Topics of interest include mathematical and scientific eLearning, learning management systems, markup languages, e.g. MathML and SVG [12], online mathematics, computer algebra systems (CAS) and dynamic geometry systems (DGS), technologies for eContent

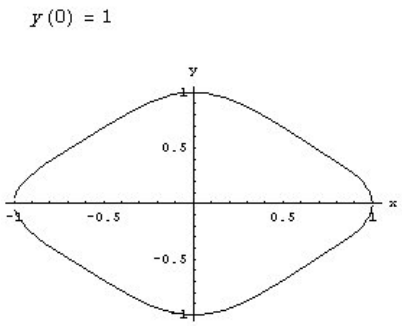

Figure 1. Graphics, Steplet for exact step-by-step solution of the differnetial equation $x^3+3y^2x+(y^3+3x^2y)y'=0$

development, integration of communities and transdisciplinary research, methodology and educational research design, software development standards and user studies.

The workshop in 2008 focuses on mathematical and scientific eContent in the educational sector with the aim of bringing together experts in technologies, mathematics and science education, practitioners and providers of tools and content. About 50 papers are accepted for talks and hands-on sessions. The 4th JEM Workshop and the 1st GeoGebra Day will be satellite events.

3. THE DMATH PROJECT

The dMath [13] project is a follow up from Xmath and was also coodinated by Buskerud University College. It was connected to the EU Leonardo Action [14] and lasted for 3 years (2003-2006). The main purpose was to build a European Database of Mathematical eLearning Modules. The database content consists of reusable learning objects (RLO) using MathML and Steplets. The system is divided into four main parts:

1. The General Authoring Suite to be used by editors.
2. dMathArchive, the repository for RLO's
3. Downloading facilities to a LMS (learning management system) for publishers
4. The Xmath eBook

The modules are organized according to standard textbook levels and a module is divided into different independent objects for downloading by a publisher. The publisher may freely use these objects in a web-based course in mathematics. The main target groups are schools on different levels and industrial companies wanting to give employees further education. This database may be used as a mathematical dictionary as well and is evolutionary in the sense that people may put own objects into the database (subjected to review). The Xmath eBook is online and may be used for supporting mathematical calculations connected to the database modules.

The editor SciWriter [15] for authoring scientific articles and mathematical or technical eLearning content was developed in dMath (LaTEX, XML/MathML, HTML, PDF).

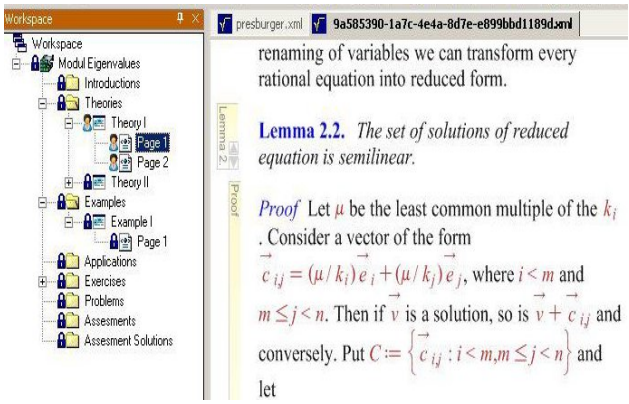

Figure 2 dMath Database, Module Eigenvalues in client workspace

Many of the participants in the dMath project are either from engineering universities or technical companies. The selection of modules is then dominated by technical and engineering studies and applications.

The tailoring of the structure of different university studies to the European Higher Education Area (EHEA), an objective forthcoming from the Bologna Declaration, is the major task outstanding in Europe's different university systems. The reform affects not only the structure of university studies, but furthermore leads to European-wide reflection on the suitability of the syllabuses in mathematics subjects and, of greater importance, on the manner in which mathematics is taught. It is now no longer possible to uphold the same approach to teaching mathematics as fifty years ago and classrooms should reflect the technological revolution that has occurred in recent decades.

In view of the characteristics of the EHEA and the aforementioned demands on teachers, certain changes, at times of a drastic nature, are required in our traditional approach to teaching. The changes required does not only affect syllabuses, stemming from weaker initial grounding, but in general affect the knowledge of mathematics acquired in the stages leading up to a university education. This is highlighted by different European-wide reports, such as the PISA report, and there is also a need to embrace a methodological change that which will allow for addressing the students' new learning needs. The use of the entire potential provided by new technologies applied to teaching may lead to reconsideration of the organisation of different teaching groups, adapting their size to the possibilities of the different computer rooms.

The Bologna declaration has started a necessary revision of the contents and of the teaching techniques to use in the teaching of mathematics at the different University levels. In this sense, the new demands created by the necessary European harmonisation and the need to have available tools that may make it possible a continuous mathematical formation in diverse learning situations, demand a new strategic response. In order to begin to give an answer to these new challenges the dMath project has been developed.

4. THE EVLM PROJECT

EVLM [16] is the European Virtual Laboratory of Mathematics. The project is a EU Leonardo project lasting from 2006-2008 and is coordinated by The Slovak Technical University [17]. The aim of EVLM is to promote better understanding and utilisation of mathematical knowledge in a range of other disciplines that are underpinned by mathematics.

Each National Centre will host a portal (in the respective national language) providing a virtual database of mathematical resources and eLearning materials available from the partner institutions and other sources (such as previously EU funded projects).

The project will promote eLearning in Mathematics, and provide solutions for different target groups and help for teachers and trainers to enhance their skill in using the most advanced educational tools and environments. It will develop an authorative catalogue of available educational materials in mathematics and establish a central database providing

information on the availability of and links to the above resources.

Further the project will privide expert consultancy on the use of these materials and increase the dissemination and utilization of these resources through translation into relevant languages (multilingual and multicultural). The opportunity for educational institutions to share special educational materials developed for their own specific purposes (frequently with funding from European programmes) with other European institutions is important.

The primary target groups are students from secondary school up to PhD, secondary school teachers, trainers, university lecturers, researchers or scientists who need to improve their knowledge and understanding of mathematics, or need expert consultancy in solving their mathematical problems. Targeted sectors are educational institutions from secondary schools to universities.

Secondary target groups are any interested party from the non-academic sphere, from industry or research and development, who need a deeper knowledge of mathematics - including the latest results and details of available information - or who need help with the solution of specific mathematical problems. Targeted sectors are research and training organisations, university enterprise training partners, research centres and scientific institutes.
A third group of potential users of the project´s results may be any training organisations, schools and university training organisations providing continuous education and distance learning or life-long learning. Potential final users are also private individuals as home self-learners, interested in self-education who, for personal reasons, may not normally engage with formal education including disabled people and people perceiving discrimination due to social and gender stereotypes.

5. THE WEBALT PROJECT

The WebAlt project [18], Web Advanced Learning Technologies, was a EU eContent project coordinated by the University of Helsinki [19] lasting from 2005 to 2007.

Under the heading of “OpenMath”, the EU has supported the development of both OpenMath [20] and MathML, the current standards for communicating semantically rich mathematics via XML. These standards are now supported by commercial software packages such as Word and PowerPoint together with MathType, and web browsers such as Internet Explorer and Mozilla. Hence MathML and OpenMath are now ready for their prime time.

The WebALT project showcases a significant application that combines existing standards for representing mathematics and existing linguistic technologies in order to enable the creation of language-independent mathematical content for multilingual and multicultural localization.

The core deliverable of WebALT is the the WebALT grammar for mathematics. Mathematical text conforming to the WebALT grammar can be automatically generated in many European languages. This means that valuable educational content can be easily located and used across Europe. Authors in Norway can now write educational content that is immediately usable in Spain, and vice versa, without labour intensive manual translation. This will supprort the Bologna process and makes it possible for educational institutions across Europe to use, for example, the same test banks.

The highlights of deliverables also includes the TextMathEditor [21] for authoring mathematical text conforming to the WebALT grammars, the WebALT eRepository (WALTER), the WebALT Editor (WExEd) [22] supporting the creation of language independent problems in mathematics, the WebALT Maple T.A FireFox plug-in and multilingual mathematics laboratories for calculus.

6. THE JEM PROJECT

The JEM project [23], Joining Educational Mathematics, is an ongoing EU eContent project coordinated by the University of Helsinki lasting from 2006 to 2009. JEM is a thematic network.
The network intends to bring together the markup technologies developer community with the user community and in particular with the authors of eLearning content in mathematics. Developers of mathematical markup standards and tools will obtain feedback from key users of the technologies and vise versa. Users of state-of-the-art web languages will have access to in-depth training and be able to influence future developments.

The overall goal of the network is to become a focal reference point for all mathematical eContent stakeholders. Further to foster and extend the multicultural and multilingual use of semantic markup for mathematics within the European eLearning community, to monitor the development of learning technologies so that

mathematical content can be integrated and delivered by state-of-the-art and learning systems and finally to structure, coordinate and support the user and authoring community of eLearning content in mathematics

7. CONCLUSIONS

Mathematical content is one of the corner stones of the information society, for education just as much as for science, technology and business. Advances by European projects in representing mathematics on the web enable content actors to create and deliver quality mathematical content to users across Europe, who need it in their work, studies or at home.

Mathematics is a particularly suitable area for electronic learning. With present day information technology it is possible to create interactive learning tools which themselves know an impressive amount of mathematics. This is to a large extent due to the exact and a priori nature of mathematical knowledge. Many areas of the exact sciences rely heavily on basic mathematical knowledge.

European projects and technology have played a fundamental role in the development of the world wide web and mathematical eLearning. The same has been true for enabling semantically rich representations of mathematical content on the web. Mathematical eLearning will also have great pedagogical advantages compared to traditional learning methods.

The real challenge of eLearning is to produce content that brings a general improvement in the way students learn and the way teachers teach. But the design and production of high-quality digital content has turned out to be harder than expected, the main reason being that it requires the convergence of many kinds of experts in parallel to the convergence of the several technologies involved. In the case of mathematics, the least amount of expertise asks for the presence of professional mathematicians, software engineers and perhaps learning theorists that can productively talk to each other. To channel this expertise in order to enhance the quality of eContent in mathematics is the main goal of the further effort.

8. ACKNOWLEDGMENT

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